Unit 2: Differentiation
2.6 Position, Velocity, & Acceleration

Higher Order Derivatives:

So far we have found *first derivatives*. We can differentiate a first derivative to obtain the *second derivative* and so on.

ex. $y = f(x) = 2x^5 + x^3$

1st derivative: $\frac{dy}{dx} = f'(x) = 10x^4 + 3x^2$

2nd derivative: $\frac{d^2y}{dx^2} = f''(x) = 40x^3 + 6x$

3rd derivative: $\frac{d^3y}{dx^3} = f'''(x) = 120x^2 + 6$

4th derivative: $\frac{d^4y}{dx^4} = f^{(4)}(x) = 240x$

5th derivative: $\frac{d^5y}{dx^5} = f^{(5)}(x) = 240$

Position, Velocity, & Acceleration

Given a position function $s(t)$, the velocity function $v(t)$ will be the 1st derivative and the acceleration function $a(t)$ will be the 2nd derivative.

ie. $v(t) = s'(t)$
$a(t) = v'(t) = s''(t)$

Position, velocity, and acceleration are all *vector* quantities (having magnitude and direction).

note: speed is the absolute value of velocity

ex. A particle's position is given by $s(t) = 12t^3 - 2t^2 + 4t$

then $v(t) = 36t^2 - 4t + 4$

and $a(t) = 72t - 4$

Horizontal motion:

Left – negative velocity
Right – positive velocity
Speed increasing – velocity & acceleration have same sign (both + or both -)
Speed decreasing – velocity & acceleration have opposite signs (oppose each other)
Direction change – $v = 0$ & $a \neq 0$

ex. The position of a particle beginning at time $t = 0$ is given by $s(t) = t^3 - 12t^2 + 36t + 18$.

a) When does the particle change direction?
b) On what time intervals is the particle moving left or right?
c) When is the particle speeding up or slowing down?
d) What is the total distance travelled by the particle between $t = 0$ and $t = 6$?

a) find $v(t) = 0$

$$v(t) = 3t^2 - 24t + 36 = 0$$
$$3(t^2 - 8t + 12) = 0$$
$$3(t-2)(t-6) = 0$$
$$t = 2 \ \ or \ \ t = 6$$

check $a(t)$: $\quad a(t) = 6t - 24$

$a(2) = -12 \neq 0 \qquad a(6) = 12 \neq 0$

Thus, the particle changes direction at $t = 2$ and $t = 6$.

b) Since the particle changes direction at times 2 and 6, split up time into intervals and test velocity in each interval:

$(0, 2)$	$(2, 6)$	$(6, \infty)$
$v(1) = 3(1-2)(1-6) > 0$	$v(3) = 3(3-2)(3-6) < 0$	$v(7) = 3(7-2)(7-6) > 0$

Thus, the particle is moving right on the time intervals $(0, 2)$ and $(6, \infty)$. The particle is moving left on $(2, 6)$.

c) The times at which speed changes are determined be analyzing when acceleration changes.

$$a(t) = 6t - 24 = 0$$
$$t = 4$$

So acceleration changes at time 4. From part A we already know that acceleration is negative before time 4 ($a(2) = -12$) and positive after time 4 ($a(6) = 12$).

Thus, the particle is slowing down on $(0, 2)$ and $(4, 6)$ since v & a are in opposite directions (opposite signs).

The particle is speeding up on $(2, 4)$ and $(6, \infty)$ since v & a are in the same direction (same sign).

d) Total distance $\neq s(6) - s(0)$ because of the turnaround at time 2. (difference between displacement and distance).

Thus, total distance = distance from 0 to 2 + distance from 2 to 6:

$$d = |s(2) - s(0)| + |s(6) - s(2)|$$
$$= |50 - 18| + |18 - 50|$$
$$= 32 + 32$$
$$= \boxed{64}$$

$$s(0) = (0)^3 - 12(0)^2 + 36(0) + 18 = 18$$
$$s(2) = (2)^3 - 12(2)^2 + 36(2) + 18 = 5$$
$$s(6) = (6)^3 - 12(6)^2 + 36(6) + 18 = 18$$

Practice Questions

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Worksheet