APCalc Date:_____
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Unit 2: Differentiation
2.5 The Chain Rule

The Chain Rule – for composite functions

If $h(x) = f(g(x))$, then

$$\boxed{\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'(g(x)) \cdot g'(x)}$$

ie. the derivative of the outer function times the derivative of the inner function

ex. $y = (x^2 + 1)^3$

$$\frac{dy}{dx} = 3(x^2 + 1)^2 \cdot \frac{d}{dx}[x^2 + 1]$$
$$= 3(x^2 + 1)^2(2x)$$
$$= \boxed{6x(x^2 + 1)^2}$$

ex. $f(x) = \sqrt{3x^2 - 2}$

rewrite: $f(x) = (3x^2 - 2)^{1/2}$

$$f'(x) = \frac{1}{2}(3x^2 - 2)^{-1/2} \cdot \frac{d}{dx}[3x^2 - 2]$$
$$= \frac{1}{2}(3x^2 - 2)^{-1/2} \cdot (6x)$$
$$= \boxed{\frac{3x}{\sqrt{3x^2 - 2}}}$$

ex.
$f(x) = \dfrac{-2}{(2x^3 + x)^2}$
rewrite:
$f(x) = -2(2x^3 + x)^{-2}$

$$f'(x) = 4(2x^3 + x)^{-3}(6x^2 + 1)$$
$$= \boxed{\frac{4(6x^2 + 1)}{(2x^3 + x)^3}}$$

ex. $y = \sin(x^2 + 2)$

$$\frac{dy}{dx} = \cos(x^2 + 2)\,(2x)$$
$$= \boxed{2x\cos(x^2 + 2)}$$

ex. $f(x) = (x^3 + x)^5(x^2 + 2)$ * product rule first, then chain rule within

$$f'(x) = (x^3 + x)^5(2x) + (x^2 + 2)[5(x^3 + x)^4(3x^2 + 1)]$$

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$$= (x^3 + x)^4[(x^3 + x)(2x) + 5(x^2 + 2)(3x^2 + 1)]$$
$$= (x^3 + x)^4[2x^4 + 2x^2 + 15x^4 + 35x^2 + 10]$$
$$= \boxed{(x^3 + x)^4(17x^4 + 37x^2 + 10)}$$

ex. $f(x) = \cos\left(\sqrt{x^2 + 2}\right)$ * double chain rule

$$f'(x) = -\sin\left(\sqrt{x^2 + 2}\right) \cdot \left(\frac{1}{2}(x^2 + 2)^{-1/2}\right) \cdot (2x)$$
$$= \boxed{\frac{-x\sin\left(\sqrt{x^2 + 2}\right)}{\sqrt{x^2 + 2}}}$$

ex.

$f(x) = \sqrt[3]{2x + 3}(x^2 + 1)$

$= (2x + 3)^{1/3}(x^2 + 1)$

$$f'(x) = (2x + 3)^{1/3}(2x) + (x^2 + 1)\left(\frac{1}{3}(2x + 3)^{-2/3}\right)(2)$$
$$= 2(2x + 3)^{-2/3}\left[(2x + 3)x + \frac{1}{3}(x^2 + 1)\right]$$
$$= \frac{2\left(2x^2 + 3x + \frac{x^2 + 1}{3}\right)}{(2x + 3)^{2/3}} \times \frac{3}{3}$$
$$= \frac{2(6x^2 + 9x + x^2 + 1)}{3\sqrt[3]{(2x + 3)^2}}$$
$$= \boxed{\frac{2(7x^2 + 9x + 1)}{3\sqrt[3]{(2x + 3)^2}}}$$

Practice Questions

Pg. 130/ # 7 – 10, 11 – 29 (odd), 43 – 55 (odd), 61, 63, 73