Date:________________

Unit 2: Differentiation
2.4 Product & Quotient Rules

Theorem – The Product Rule

The derivative of the product fg of two differentiable functions f and g exists and

$$\boxed{\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)}$$

ex. $h(x) = (3x - 2x^2)(5 + 4x)$

$$\begin{aligned} h'(x) &= (3x - 2x^2)\frac{d}{dx}[5 + 4x] + (5 + 4x)\frac{d}{dx}[3x - 2x^2] \\ &= (3x - 2x^2)(4) + (5 + 4x)(3 - 4x) \\ &= 12x - 8x^2 + 15 - 8x - 16x^2 \end{aligned}$$

$$\boxed{h'(x) = -24x^2 + 4x + 15}$$

ex. $y = x\sin x$

$$\begin{aligned} \frac{dy}{dx} &= x\frac{d}{dx}[\sin x] + \sin x\frac{d}{dx}[x] \\ &= x\cos x + \sin x\,(1) \end{aligned}$$

$$\boxed{\frac{dy}{dx} = x\cos x + \sin x}$$

ex. $f(x) = 2x\cos x - 2\sin x$

$$\begin{aligned} f'(x) &= 2x\frac{d}{dx}[\cos x] + \cos x\frac{d}{dx}[2x] - 2\frac{d}{dx}[\sin x] \\ &= 2x(-\sin x) + \cos x\,(2) - 2\cos x \end{aligned}$$

$$\boxed{f'(x) = -2x\sin x}$$

Theorem – The Quotient Rule

The derivative of the quotient $\frac{f}{g}$ of two differentiable functions f and g exists for all x such that $g(x) \neq 0$ and

$$\boxed{\frac{d}{dx}\left[\frac{[f(x)]}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}}$$

ex.

$$f(x) = \frac{5x-2}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)\frac{d}{dx}[5x-2] - (5x-2)\frac{d}{dx}[x^2+1]}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(5) - (5x-2)(2x)}{(x^2+1)^2}$$

$$= \frac{5x^2+5-10x^2+4x}{(x^2+1)^2}$$

$$\boxed{f'(x) = \frac{-5x^2+4x+5}{(x^2+1)^2}}$$

ex.

$$y = \frac{3-\frac{1}{x}}{x+5}$$

rewrite:

$$y = \frac{3x-1}{x} \cdot \frac{1}{x+5} = \frac{3x-1}{x^2+5x}$$

$$\frac{dy}{dx} = \frac{(x^2+5x)\frac{d}{dx}[3x-1] - (3x-1)\frac{d}{dx}[x^2+5x]}{(x^2+5x)^2}$$

$$= \frac{(x^2+5x)(3) - (3x-1)(2x+5)}{(x^2+5x)^2}$$

$$= \frac{3x^2+15x-6x^2-13x+5}{(x^2+5x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2+2x+5}{(x^2+5x)^2}}$$

ex. Determine the derivative of $\tan x$

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{(\cos x)\frac{d}{dx}[\sin x] - (\sin x)\frac{d}{dx}[\cos x]}{(\cos x)^2}$$

$$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$\boxed{\frac{d}{dx}[\tan x] = \sec^2 x}$$

Similarly we can determine derivatives for the other trig ratios:

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Practice Questions

Pg. 121/ # 1 – 6, 13 – 41 (odd), 51, 53, 59 – 62