APCalc Date:________________
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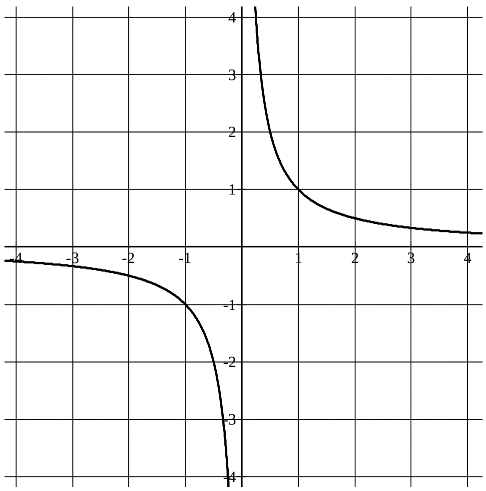
## Unit 1: Limits
## 1.6 Infinite Limits

Limits that equal $\pm\infty$ do not exist, but it is more useful to state whether they go to $\pm\infty$ (when possible) as it allows us to understand (picture) why the limits does not exist.



ex. $f(x) = \frac{1}{x}$

$\lim_{x\to 0^-} f(x) = -\infty$
ie. $f(x)$ decreases without bound as $x$ approaches 0 from the left

$\lim_{x\to 0^+} f(x) = \infty$
ie. $f(x)$ increases without bound as $x$ approaches 0 from the right

*note: neither limit actually exists but it is more informative to say they equal $\infty$ or $-\infty$ then to just say DNE

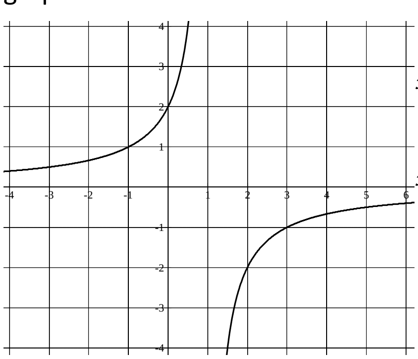
If $f(x)$ approaches infinity (or negative) infinity as $x$ approaches $c$ from the left or right, then the line $x = c$ is a vertical asymptote of the graph of $f$.

Theorem
Let $f$ and $g$ be continuous on an open interval containing $c$. If $f(c) \neq 0$, $g(c) = 0$, and $g(x) \neq 0$ for all $x \neq c$ in the interval then the graph of $h(x) = \frac{f(x)}{g(x)}$ has a vertical asymptote at $x = c$.

This is the official theorem that establishes the $\frac{0}{0}$ vs $\frac{k}{0}$ principle that we have already been using.
ie. $h(c) = \frac{k}{0}$ (where $k$ is a non-zero constant) means the limit as $x$ approaches $c$ will not exist ($= \pm\infty$), and thus $c$ is an infinite discontinuity ($x = c$ vertical asymptote)

ex. $f(x) = \frac{-2}{x-1}$ has a discontinuity at $x = 1$ since $f(1)$ is undefined.
Because $f(1) = \frac{-2}{0}$ we know there is a vertical asymptote at $x = 1$.
No limits as $x$ approaches 1 will exist, but either a graphical or numerical analysis will allow us to state whether the function increases or decreases without bound at the asymptote.

graphical:



$\lim_{x\to 1^-} f(x) = \infty$

$\lim_{x\to 1^+} f(x) = -\infty$

numerical:

$f(0.9) = \frac{-2}{-0.1} > 0$ thus increasing to $\infty$

$f(1.1) = \frac{-2}{0.1} < 0$ thus decreasing to $-\infty$

note: if the both one sided limits are equal to $\infty$ (or both equal to $-\infty$) then we can state a single limit as equalling $\infty$ (or $-\infty$)

Properties of Infinite Limits
Let $c$ and $L$ be real numbers and let $f$ and $g$ be functions such that $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$, then:

1) Sum or difference: $\lim_{x \to c}[f(x) \pm g(x)] = \infty \pm L = \infty$
2) Product: $\lim_{x \to c}[f(x) \cdot g(x)] = \infty \cdot L = \begin{cases} \infty, L > 0 \\ -\infty, L < 0 \end{cases}$
3) Quotient: $\lim_{x \to c} \frac{g(x)}{f(x)} = \frac{L}{\infty} = 0$

We can also apply these properties to one-sided limits and to $\lim_{x \to c} f(x) = -\infty$

ex. $\lim_{x \to 0} \left(\cos x + \frac{1}{x^2}\right) = 1 + \infty = \boxed{\infty}$

Practice Questions

Pg. 84 / # 1 – 4, 5, 7, 9 – 14, 19, 25 – 43 odd, 55 - 58