APCalc Date:________________
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Unit 1: Limits
1.5 Continuity II & The Intermediate Value Theorem

ex. What value of b will make $f(x) = \begin{cases} 2x+b, & x<0 \\ \cos x, & x \geq 0 \end{cases}$ everywhere continuous?

The pieces $2x+b$ and $\cos x$ are everywhere continuous so need to make sure they "connect" at $x=0$.
Use the 3 part definition of continuity at a point:

1) $f(c)$ is defined

$$f(0) = \cos(0) = 1 \qquad \text{yes}$$

2) $\lim_{x \to c} f(x)$ exists

$$\lim_{x \to 0^-} f(x) = 2(0) + b = b \qquad\qquad \lim_{x \to 0^+} f(x) = \cos(0) = 1$$

$$\therefore \lim_{x \to 0} f(x) \text{ exists iff } b = 1$$

3) $\lim_{x \to c} f(x) = f(c)$

$$\lim_{x \to 0} f(x) = f(0) \text{ iff } b = 1$$

Thus, $\boxed{b = 1}$

Properties of Continuity

If f and g are continuous at $x=c$ then the following are also continuous at $x=c$:

1) $k \cdot f(x)$ where k is a real number constant
2) $f(x) \pm g(x)$
3) $f(x) \cdot g(x)$
4) $\frac{f(x)}{g(x)}$; $g(c) \neq 0$

ex. $f(x) = \frac{\sin x}{x^2+1}$ will be everywhere continuous because $\sin x$ and x^2+1 are everywhere continuous

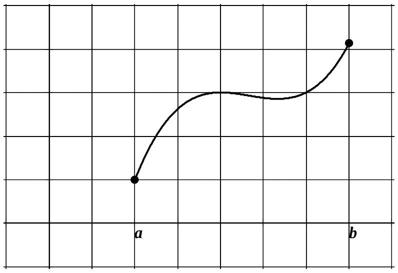
ex. $f(x) = \log x + x^3$ will be everywhere continuous **on its domain** ($x > 0$) because $\log x$ and x^3 are everywhere continuous on their domains

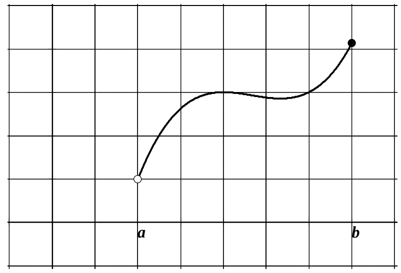
Definition of Continuity on a Closed Interval

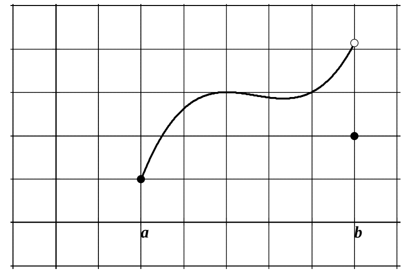
A function f is continuous on $[a, b]$ if it is continuous on (a, b) and the following one-sided limits exist and satisfy the conditions:

$$\lim_{x \to a^+} f(x) = f(a) \quad \text{and} \quad \lim_{x \to b^-} f(x) = f(b)$$

ie. the function is continuous at all points in-between the endpoints and the graph "connects" to its endpoints ex.







Continuous on (a, b)
Continuous on $[a, b]$

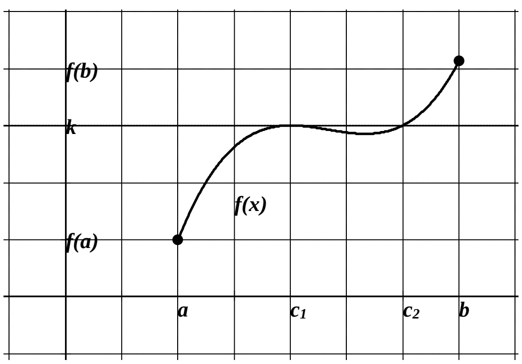
Continuous on (a, b)
NOT Continuous on $[a, b]$
$\lim_{x \to a^+} f(x) = DNE$

Continuous on (a, b)
NOT Continuous on $[a, b]$
$\lim_{x \to b^-} f(x) \neq f(b)$

Intermediate Value Theorem (IVT)

If f is continuous on $[a, b]$ and $y = k$ is any value between $f(a)$ and $f(b)$, then there exists at least one value $x = c$ in $[a, b]$ such that $f(c) = k$.

ex.



$f(x)$ is continuous on $[a, b]$

$f(a) < k < f(b)$

$\therefore$ there exists a value c in $[a, b]$ such that $f(c) = k$

In fact, in this case there are two values c_1 and c_2.

*note the importance of continuity to the IVT

ex. Show that $f(x) = x^3 + 2x + 1$ has a zero in the interval $[-3,3]$.

$f(x)$ is a polynomial function and thus is everywhere continuous, including on $[-3,3]$.

$f(-3) = (-3)^3 + 2(-3) + 1 = -32$

$f(3) = (3)^3 + 2(3) + 1 = 34$

A zero of the function means $f(x) = 0$. Since this value is between -32 and 34, the IVT guarantees that there is at least one value c on $[-3,3]$ such that $f(c) = 0$.

Practice Questions

Pg. 77 / # 55 – 58, 67 – 74, 79 – 83, 90 – 94