APCalc Date:________________

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Unit 1: Limits

1.2 Evaluating Limits Analytically

Substitution technique - If $f(x)$ is **not** a piecewise function and $f(c)$ has a real value then:

$$\lim_{x \to c} f(x) = f(c)$$

ie. in a non-piecewise function limits will equal the value of the function, provided the function is defined at the value of *c* (direct substitution will determine the limit value)

ex.

$$\lim_{x \to 2} x^2 + 4 = f(2) = (2)^2 + 4 = \boxed{8}$$

$$\lim_{x \to 0} \frac{x^3 + 2x^2 + 1}{x + 2} = \frac{(0)^3 + 2(0)^2 + 1}{(0) + 2} = \boxed{\frac{1}{2}}$$

What if $f(c)$ does not exist?

Cancellation technique – if $f(x)$ and $g(x)$ are functions that agree at all values of x except $x = c$, and $f(c)$ does not exist but $g(c)$ does, then

$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = \mathrm{g(c)}$$

ex. $f(x) = \frac{x^2-4}{x-2}$ (function we analyzed in 1.1)

The function does not exist at $x = 2$, thus we cannot evaluate the limit through direct substitution.

Through algebraic manipulation (in this case cancellation) we can create an equivalent function which will share all the same limit values and which can be evaluated at $x = 2$.

$$\frac{x^2 - 4}{x - 2}$$

$$= \frac{(x - 2)(x + 2)}{x - 2}$$

$$= x + 2$$

*this explains why the graph of $f(x)$ was a straight line except for the hole at $x = 2$

$$\therefore \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} x + 2 = (2) + 2 = \boxed{4}$$

ex. evaluate:

$$\lim_{x \to 0} \frac{2x^2 + x}{x}$$

$$= \lim_{x \to 0} 2x + 1$$

$$= 2(0) + 1$$

$$= \boxed{1}$$

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

$$= \lim_{x \to -3} \frac{(x + 3)(x - 2)}{x + 3}$$

$$= \lim_{x \to -3} x - 2$$

$$= (-3) - 2$$

$$= \boxed{-5}$$

Rationalization technique – to be used when radicals are involved (in either numerator or denominator)

ex. evaluate:

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$$

$$=\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\times\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$$ *multiply num. and den. by the conjugate

$$=\lim_{x\to 3}\frac{(x+1)-4}{(x-3)\left(\sqrt{x+1}+2\right)}$$

$$=\lim_{x\to 3}\frac{x-3}{(x-3)\left(\sqrt{x+1}+2\right)}$$

$$=\lim_{x\to 3}\frac{1}{\sqrt{x+1}+2}$$

$$=\frac{1}{\sqrt{(3)+1}+2}$$

$$=\boxed{\frac{1}{4}}$$

ex. evaluate:

$$\lim_{x\to -2}\frac{-x-2}{\sqrt{2-x}-2}$$

$$=\lim_{x\to -2}\frac{-x-2}{\sqrt{2-x}-2}\times\frac{\sqrt{2-x}+2}{\sqrt{2-x}+2}$$

$$=\lim_{x\to -2}\frac{(-x-2)\left(\sqrt{2-x}+2\right)}{(2-x)-4}$$

$$=\lim_{x\to -2}\frac{(-x-2)\left(\sqrt{2-x}+2\right)}{-x-2}$$

$$=\lim_{x\to -2}\sqrt{2-x}+2$$

$$=\sqrt{2-(-2)}+2$$

$$=\boxed{4}$$

*note: cancellation and rationalization techniques will only work to determine $\lim\limits_{x\to c} f(x)$ if $f(c)=\frac{0}{0}$

if $f(c)=\frac{k}{0}$, where k is a real value $\neq 0$, then $\lim\limits_{x\to c} f(x)$ will not exist

Practice Questions

Pg. 64/ # 5 – 27 odd, 33 – 52, 73 - 76