APCalc Date:_____
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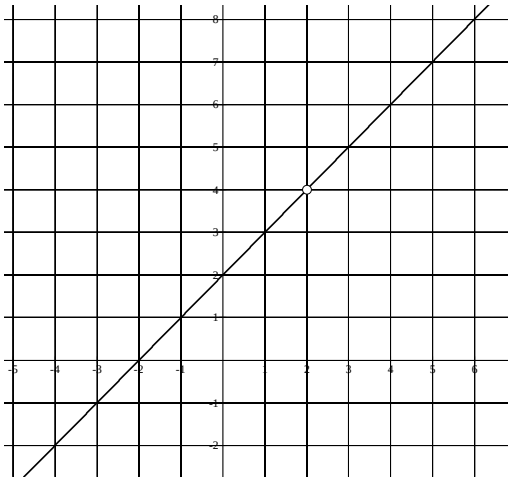
Unit 1: Limits
1.1 Evaluating Limits Graphically & Numerically

What is a limit?

An indication of where the output values of a function (ie. y-values) are heading, regardless of if they actually reach that value.

ex. Consider the function $f(x) = \frac{x^2-4}{x-2}$



*use calc. to look at the graph but be aware of domain restriction $x \neq 2$ which causes a hole in the graph

So far in math we have only evaluated functions (ie. input to output).

ex. $f(-1) = 1$ $f(3) = 5$

What if we were asked for $f(2)$?

$f(2)$ is undefined but just stating that gives no indication of how the function is behaving as it approaches this undefined point.

Thus, instead of asking for $f(2)$ we ask for the limit of $f(x)$ as x approaches 2. This is denoted:

$$\lim_{x\to 2} f(x)$$

Imagine you and a friend are tiny microscopic people travelling along the line toward the hole from different directions. You can tell each other (through your amazing calculus walkie-talkies) where you are. You might tell your friend: "I am at $x = 1$ and $y = 3$." Your friend replies: "I am at $x = 4$ and $y = 6$." These are essentially function notation statements (input to output). You are stating $f(1) = 3$, while your friend is stating $f(4) = 6$.

What do you tell each other as you near the hole? Both of you would say: "I am approaching a hole at $x = 2$ and $y = 4$." This does not mean that $f(2) = 4$, in fact $f(2)$ is still undefined, but instead it means that the limit of $f(x)$ as x approaches 2 is 4, or:

$$\lim_{x\to 2} f(x) = 4$$

So a limit is an output y-value that a function is approaching as you near a certain input x-value.

Sometimes the value of the function and the value of the limit may equal each other. In the above example that is true at all other points where $x \neq 2$.

ex.

$$f(-2) = \lim_{x\to -2} f(x) = 0$$

or

$$f(1) = \lim_{x\to 1} f(x) = 3$$

This limiting process can also be done numerically. To determine how the function behaves near $x = 2$ set up a table with values for x that approach 2 from both sides and evaluate the function at these values (a variation on a table of values).

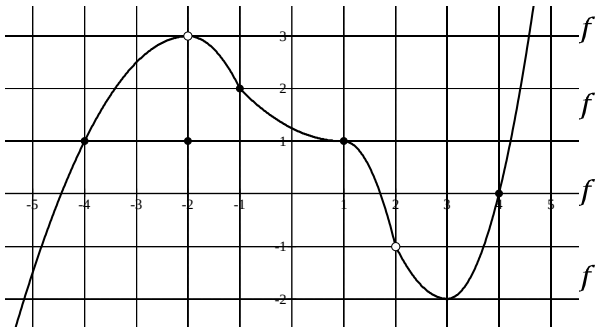
ex. $f(x) = \frac{x^2-4}{x-2}$ *hint: use calc (trace) value finder to evaluate

	x approaches 2 from the left →				← x approaches 2 from the right		
x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	3.9	3.99	3.999		4.001	4.01	4.1
	→				←		
	$f(x)$ approaches 4				$f(x)$ approaches 4		

Thus, as x approaches a value of 2 we see that $f(x)$ approaches a value of 4. In other words:

$$\lim_{x\to 2} f(x) = 4$$

ex. Given $f(x)$ shown state:



$f(-4) = 1$ $\quad \lim_{x\to -4} f(x) = 1$

$f(2) = \infty$ $\quad \lim_{x\to 2} f(x) = -1$

$f(4) = 0$ $\quad \lim_{x\to 4} f(x) = 0$

$f(-2) = 1$ $\quad \lim_{x\to -2} f(x) = 3$

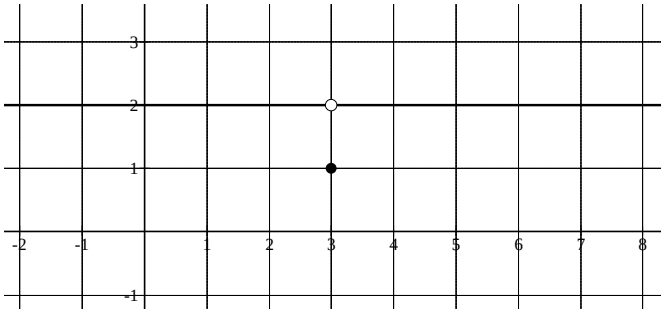
Note: Whether a function does or does not exist at a certain value of x has no impact on wether the function has a limit as x approaches that value.

ex. Given the piecewise function $f(x) = \begin{cases} 2; x \neq 3 \\ 1; x = 3 \end{cases}$ determine the limit as x approaches 3 numerically.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	2	2	2		2	2	2

$$\therefore \lim_{x\to 3} f(x) = 2$$

*Note that the fact $f(3) = 1$ has no bearing on the existence or value of the limit.
Graphically the situation would look like this:

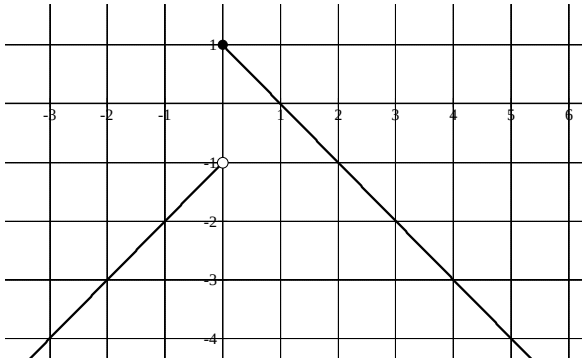


Do limits always exist?

No, there are three conditions where a limit does not exist:

1) Behaviour that differs from left & right.

ex. $f(x) = \begin{cases} x - 1; x < 0 \\ -x + 1; x \geq 0 \end{cases}$



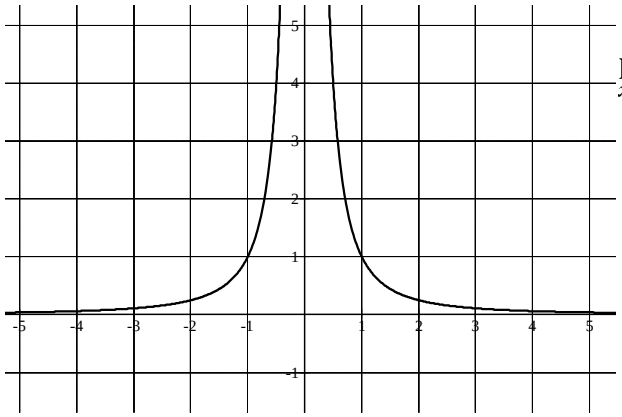
$\lim_{x \to 0} f(x) = DNE$

is undefined because $f(x)$ approaches different values as x approaches 0 from each direction. A limit must equal the same real value from each direction to exist.

Thus, the limit does not exist (DNE).

2) Unbounded behaviour (asymptotic behaviour).

ex. $f(x) = \frac{1}{x^2}$



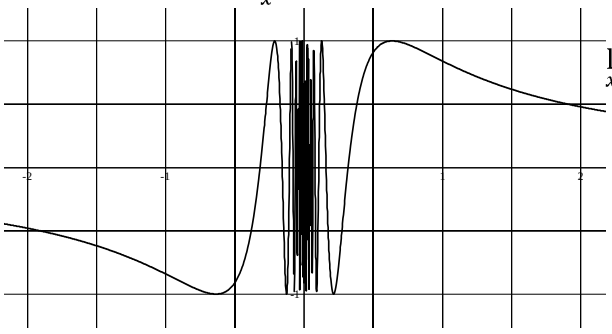
$\lim_{x \to 0} f(x) = DNE$

is undefined because $f(x)$ approaches infinity as x approaches 0 from each direction. A limit must equal a real value to exist.

Thus, the limit does not exist (DNE).

3) Oscillating behaviour.

ex. $f(x) = \sin\frac{1}{x}$



$\lim_{x \to 0} f(x) = DNE$

is undefined because $f(x)$ jumps between 1 and -1 as x approaches 0 from each direction.

Thus, the limit does not exist (DNE).

Practice Questions

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